



WORKSHEET

The Binomial Distribution Question

Definition of a Binomial Variable

A **random variable** X is a binomial variable if the following conditions are satisfied:

1. Each trial is **independent** of the others.
2. There is a fixed number of trials.
3. The outcome of each trial can be classified as either success or failure.
4. The probability of success stays constant over each trial.

Random Variable: A variable whose value depends on chance in some way.

Independent Trail: Trial is independent when the outcome of previous trials does not affect the result of the current trial.

Definition of the Binomial Distribution

If random variable X is a binomial variable, then X has a binomial distribution:

$$P(X = x) = \binom{n}{x} p^x q^{n-x}$$

Where:

n = Number of trials

p = Probability of success

$q = 1 - p$ = Probability of failure

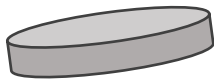
$P(X = x)$ = Probabilitiy we get x successes in n trials

Mean and Variance of the Binomial Distribution

$$\text{Mean}(X) = np$$

$$\text{Var}(X) = npq = np(1 - p)$$

Example of a Binomial Variable



Biased Coin:

- $P(\text{Head}) = 0.75$
- $P(\text{Tail}) = 0.25$

X = The number of heads after 5 flips of the coin

Random Variable **X**:

- Has a **fixed number of trials** ($n = 5$)
- The trials are **independent** as the probability of flipping a head does not depend on the result of the previous flips.
- **Success** is classified as 'flipping a head' and **failure** is classified as 'not flipping a head' or 'flipping a tail'.
- **Constant probability** of success i.e. $P(\text{Head}) = 0.75$ over all trials.

Therefore the random variable **X** is a **Binomial Variable**

Logarithm Power Rule

$$\text{Log}_b(x^n) = n\text{Log}_b(x)$$

QUESTIONS

Basic

This question type shows you how to use the Binomial Distribution Formula in a simple way to compute basic probability calculations. They ask something in this way Every Exam, so this is basically free marks.

1

A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

2

A statistics student asks people to complete a survey. The probability that a randomly chosen person agrees to complete the survey is 0.2. Find the probability that at least one of the first three people asked agrees to complete the survey. [2]

3

In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

4

(i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

5

On any day at noon, the probabilities that Kersley is asleep or studying are 0.2 and 0.6 respectively.

(i) Find the probability that, in any 7-day period, Kersley is either asleep or studying at noon on at least 6 days. [3]

6

Blank CDs are packed in boxes of 30. The probability that a blank CD is faulty is 0.04. A box is rejected if more than 2 of the blank CDs are faulty.

(i) Find the probability that a box is rejected. [3]

7

On average, 34% of the people who go to a particular theatre are men.

- (i) A random sample of 14 people who go to the theatre is chosen. Find the probability that at most 2 people are men. [3]

8

In a certain country, 60% of mobile phones sold are made by Company *A*, 35% are made by Company *B* and 5% are made by other companies.

- (i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company *A*. [3]

9

At the Nonland Business College, all students sit an accountancy examination at the end of their first year of study. On average, 80% of the students pass this examination.

- (i) A random sample of 9 students who will take this examination is chosen. Find the probability that at most 6 of these students will pass the examination. [3]

10

In a certain country the probability that a child owns a bicycle is 0.65.

- (i) A random sample of 15 children from this country is chosen. Find the probability that more than 12 own a bicycle. [3]

'Smallest Value of n '

This common question has the same structure every time it is asked in exams. They always ask you to find 'the smallest value of n ' using your binomial distribution formula. Complete this section to understand how to answer this question. (You will see how similar the answers are...)

1

A factory makes water pistols, 8% of which do not work properly.

- (i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]
- (ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n . [3]

2

When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than \$50 and 13% spend at least \$50.

- (i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]
- (ii) n people visiting the shop are chosen at random. The probability that none of them spends at least \$50 is less than 0.04. Find the smallest possible value of n . [3]

3

Eggs are sold in boxes of 20. Cracked eggs occur independently and the mean number of cracked eggs in a box is 1.4.

- (i) Calculate the probability that a randomly chosen box contains exactly 2 cracked eggs. [3]
- (ii) Calculate the probability that a randomly chosen box contains at least 1 cracked egg. [2]
- (iii) A shop sells n of these boxes of eggs. Find the smallest value of n such that the probability of there being at least 1 cracked egg in each box sold is less than 0.01. [2]

4

The probability that Janice will buy an item online in any week is 0.35. Janice does not buy more than one item online in any week.

- (i) Find the probability that, in a 10-week period, Janice buys at most 7 items online. [3]
- (ii) The probability that Janice buys at least one item online in a period of n weeks is greater than 0.99. Find the smallest possible value of n . [3]

5

Annan has designed a new logo for a sportswear company. A survey of a large number of customers found that 42% of customers rated the logo as good.

- (i) A random sample of 10 customers is chosen. Find the probability that fewer than 8 of them rate the logo as good. [3]
- (ii) On another occasion, a random sample of n customers of the company is chosen. Find the smallest value of n for which the probability that at least one person rates the logo as good is greater than 0.995. [3]

Probability Mix

This section shows you how they mix binomial distribution probability calculations with other probability calculations. It will teach you how to identify when to use the binomial distribution formula and when to not.

1

Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

- (i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]
- (ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

2

Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

- (i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]
- (ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]
- (iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

3

A fair triangular spinner has three sides numbered 1, 2, 3. When the spinner is spun, the score is the number of the side on which it lands. The spinner is spun four times.

- (i) Find the probability that at least two of the scores are 3. [3]
- (ii) Find the probability that the sum of the four scores is 5. [3]

4

Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

- (i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]
- (ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

5

Jake attempts the crossword puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75, independently of all other days.

- (i) Find the probability that he will complete the puzzle at least three times over a period of five days. [3]

Kenny also attempts the puzzle every day. The probability that he will complete the puzzle on a Monday is 0.8. The probability that he will complete it on a Tuesday is 0.9 if he completed it on the previous day and 0.6 if he did not complete it on the previous day.

- (ii) Find the probability that Kenny will complete the puzzle on at least one of the two days Monday and Tuesday in a randomly chosen week. [3]

6

An experiment consists of throwing a biased die 30 times and noting the number of 4s obtained. This experiment was repeated many times and the average number of 4s obtained in 30 throws was found to be 6.21.

- (i) Estimate the probability of throwing a 4. [1]

Hence

- (ii) find the variance of the number of 4s obtained in 30 throws, [1]

- (iii) find the probability that in 15 throws the number of 4s obtained is 2 or more. [3]

7

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X .

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

8

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

- (i) Find the probability that Khalid hurts himself on any particular day. [2]
- (ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]
- (iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]
- (iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

ANSWERS

Basic

$$(1) \quad P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$n=10$ $X = \#$ of sixes rolled on a fair dice.

$p = \text{probability of success}$
 $= \text{probability of rolling a six}$
 $= 1/6$

$$\therefore \boxed{p = 1/6}$$

$$\begin{aligned} P(3 \leq X \leq 5) &= P(X=3) + P(X=4) + P(X=5) \\ &= \binom{10}{3} (1/6)^3 (5/6)^7 + \\ &\quad \binom{10}{4} (1/6)^4 (5/6)^6 + \\ &\quad \binom{10}{5} (1/6)^5 (5/6)^5 \\ &= 0.222 \end{aligned}$$

$\therefore$ When rolling a fair dice 10 times the probability of rolling a six 3, 4 or 5 times out of 10 rolls:

$$\boxed{P(3 \leq X \leq 5) = 0.222}$$

$$(2) \quad p(\text{success}) = \text{Person agrees to complete survey} = 0.2$$

$$p(\text{failure}) = 1 - p(\text{success}) = 0.8$$

X = # of randomly selected people who complete the survey.

$n = 3$ (3 random people asked to complete survey)

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$\uparrow$ $p(\text{success})$ $\uparrow$ $p(\text{failure})$

∴ Probability at least

1 of the 3 people complete the survey = $P(X \geq 1)$

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$$

OR SIMPLER WAY

$$\begin{aligned} P(X \geq 1) &= 1 - P(X=0) \\ &= 1 - \binom{3}{0} (0.2)^0 (0.8)^3 \end{aligned}$$

$$\therefore \boxed{P(X \geq 1) = 0.488}$$

(3) $p(\text{success}) = \text{probability a car is fitted with satellite navigation.}$

$$= 0.76$$

$$n = 11$$

$X = \# \text{ of cars fitted with satellite navigation.}$

Probability that fewer than 10 cars fitted out of 11 randomly selected cars.

$$= P(X < 10)$$

$$\begin{aligned}\therefore P(X < 10) &= 1 - [P(X=10) + P(X=11)] \\ &= 1 - \left[\binom{11}{10} (0.76)^{10} (0.24)^1 + \binom{11}{11} (0.76)^{11} (0.24)^0 \right] \\ &= 0.781\end{aligned}$$

$$\therefore \boxed{P(X < 10) = 0.781}$$

$$(4) \quad p(\text{success}) = \text{Probability house has a printer.} \\ = 0.68$$

$$n = 8 \quad X = \# \text{ of houses that have printers.}$$

$$\begin{aligned} P(5, 6, 7) &= P(X=5) + P(X=6) + P(X=7) \\ &= \binom{8}{5} (0.68)^5 (0.32)^3 + \binom{8}{6} (0.68)^6 (0.32)^2 \\ &\quad + \binom{8}{7} (0.68)^7 (0.32)^1 \\ &= 0.722 \end{aligned}$$

$$\therefore \boxed{P(5, 6, 7) = 0.722}$$

$$(5) \quad p(\text{asleep}) = 0.2, \quad p(\text{study}) = 0.6$$

$$\begin{aligned} p(\text{success}) &= \text{Probability Kersley is} \\ &\quad \text{asleep or studying} \\ &= p(\text{asleep}) + p(\text{study}) \\ &= 0.2 + 0.6 = 0.8 \end{aligned}$$

$$p(\text{success}) = 0.8$$

$$n = 7 \quad X = \# \text{ of days Kersley is} \\ \text{asleep or studying @} \\ \text{noon.}$$

$$\begin{aligned} p(X \geq 6) &= P(X=6) + P(X=7) \\ &= \binom{7}{6} (0.8)^6 (0.2) + \binom{7}{7} (0.8)^7 (0.2)^0 \\ &= (7) (0.8)^6 (0.2) + (0.8)^7 \\ &= 0.577 \end{aligned}$$

$$\therefore \boxed{P(X \geq 6) = 0.577}$$

$$(6) \quad P(\text{success}) = \text{Probability a CD is faulty.} \\ = 0.04$$

Probability a box is rejected = Probability that more than 2 CD's are faulty out of 30. (# CD's in a box)

$$\therefore P(\text{Box Rejected}) = P(X > 2)$$

where $X = \# \text{ of faulty CD's.}$

$$n = 30$$

$$\therefore P(X > 2) = 1 - P(X \leq 2) \\ = 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$\therefore P(X > 2) = 1 - \left[\binom{30}{0} (0.04)^0 (0.96)^{30} + \binom{30}{1} (0.04)^1 (0.96)^{29} + \binom{30}{2} (0.04)^2 (0.96)^{28} \right]$$

$$= 0.117$$

$$\therefore \boxed{P(\text{Box Rejected}) = P(X > 2) = 0.117}$$

$$(7) \quad p(\text{success}) = \text{If person that goes to the theatre is a man} \\ = 0.34$$

Random Sample : $n=14$ $X = \#$ of people that go to the theatre that are men.
Size

Probability that at most 2 of the 14 people are men $= P(X \leq 2)$

$$\begin{aligned} \therefore P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= \binom{14}{0} (0.34)^0 (0.66)^{14} + \\ &\quad \binom{14}{1} (0.34)^1 (0.66)^{13} + \\ &\quad \binom{14}{2} (0.34)^2 (0.66)^{12} \\ &= 0.0963 \end{aligned}$$

$$\therefore \boxed{P(X \leq 2) = 0.0963}$$

$$(8) \quad p(A) = 0.6, \quad p(B) = 0.35, \quad p(\text{other}) = 0.05$$

$$n = 13$$

$X = \#$ of phones
made by company
A.

$$p(\text{success}) = p(A) = \text{Probability phone made by company A.}$$
$$\therefore p(\text{success}) = 0.6$$

$$p(X < 11) = 1 - p(X \geq 11)$$
$$= 1 - [p(X=11) + p(X=12) + p(X=13)]$$

$$\therefore p(X < 11) = 1 - \left[\binom{13}{11} (0.6)^{11} (0.4)^2 + \binom{13}{12} (0.6)^{12} (0.4)^1 + \binom{13}{13} (0.6)^{13} (0.4)^0 \right]$$
$$= 0.942$$

$$\therefore \boxed{p(X < 11) = 0.942}$$

(9) $P(\text{success}) = \text{Probability a student passes.}$
[(If you're doing this WORKSHEET it's 100%.)]
😊

$$\therefore P(\text{success}) = P(\text{passing}) = 0.8$$

$$n=9$$

$X = \#$ of students who pass.

Probability at
the most 6
students will
pass

$$= P(X \leq 6)$$

$$\begin{aligned}\therefore P(X \leq 6) &= 1 - P(X > 6) \\ &= 1 - [P(X=7) + P(X=8) + P(X=9)] \\ &= 1 - \left[\binom{9}{7} (0.8)^7 (0.2)^2 + \right. \\ &\quad \left. \binom{9}{8} (0.8)^8 (0.2) + \right. \\ &\quad \left. \binom{9}{9} (0.8)^9 (0.2)^0 \right] \\ &= 0.262\end{aligned}$$

$$\therefore \boxed{P(X \leq 6) = 0.262}$$

(10) $p(\text{success}) = \text{Probability child owns a bicycle.}$

$$\therefore p(\text{Success}) = 0.65$$

$n = 15$, $X = \# \text{ of children that own bicycles.}$

$$\begin{aligned} P(X > 12) &= P(X=13) + P(X=14) + P(X=15) \\ &= \binom{15}{13} (0.65)^{13} (0.35)^2 + \\ &\quad \binom{15}{14} (0.65)^{14} (0.35)^1 + \\ &\quad \binom{15}{15} (0.65)^{15} (0.35)^0 \\ &= 0.0617 \end{aligned}$$

$$\therefore \boxed{P(X > 12) = 0.0617}$$

'Smallest Value of n'

(1) (i) $P(\text{success}) = \text{Probability Pistol doesn't work}$

$$\therefore p(s) = 0.08$$

$$n = 19$$

$X = \#$ of pistols which don't work properly.

$$\begin{aligned} P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= \binom{19}{0} (0.08)^0 (0.92)^{19} + \\ &\quad \binom{19}{1} (0.08) (0.92)^{18} + \\ &\quad \binom{19}{2} (0.08)^2 (0.92)^{17} \end{aligned}$$

$$\therefore \boxed{P(X \leq 2) = 0.809}$$

(ii) $P(X \geq 1) > 0.9$ for n pistols.

$$\therefore 1 - P(X=0) > 0.9$$

$$1 - \binom{n}{0} (0.08)^0 (0.92)^n > 0.9$$

$$1 - (0.92)^n > 0.9$$

$$-(0.92)^n > -0.10$$

$$0.10 > (0.92)^n$$

$$\ln(0.1) > \ln(0.92)^n$$

(Showing
you every
step here)

$$\therefore \ln(0.92)^n < \ln(0.1)$$

$$n \ln(0.92) < \ln(0.1)$$

$$-0.0834n < -2.303$$

$$-0.0834n + 2.303 < 0$$

$$\therefore 2.303 < 0.0834n$$

$$\therefore 0.0834n > 2.303$$

$$\therefore n > 27.6$$

$\therefore$ Smallest possible value of n :

$$\boxed{n = 28}$$

$$(2)(i) \quad P(\text{not buy}) = 0.34 \quad P(\text{Buy something}) = 0.53 + 0.13 = 0.66$$

$$P(< \$50) = 0.53$$

$$P(\geq \$50) = 0.13 \quad \therefore P(\text{success}) = 0.66$$

$$n = 15 \quad X = \# \text{ of people that buy something.}$$

$$P(S) = 0.66$$

$$P(X \geq 14) = P(X=14) + P(X=15)$$

$$= \binom{15}{14} (0.66)^{14} (0.34) + (0.66)^{15}$$

$$\therefore \boxed{P(X \geq 14) = 0.0171}$$

$$(ii) \quad P(\geq \$50) = 0.13 \quad \therefore P(S) = 0.13$$

$X = \# \text{ of people that spend at least } \$50.$

$$\begin{array}{l} \text{Probability no one} \\ \text{spends at least} \\ \$50 \end{array} = P(X=0)$$

$$\therefore P(X=0) < 0.04$$

$$\binom{n}{0} (0.13)^0 (0.87)^n < 0.04$$

$$(0.87)^n < 0.04$$

$$\ln((0.87)^n) < \ln(0.04)$$

$$n \ln(0.87) < \ln(0.04)$$

$$-0.139n < -3.219$$

$$\therefore n > 23.2$$

$\therefore$ smallest value of n :

$$\boxed{n = 24}$$

(3)(i) $n=20$ $\mu=1.4$ $X = \# \text{ of cracked eggs in a box.}$

$$P(X=2) = \binom{20}{2} p^2 (1-p)^{18}$$

$p = P(\text{success}) = \text{probability that a randomly selected box has a cracked egg.}$

For Binomial distribution:

$$\mu = np$$

$$\therefore p = \frac{\mu}{n} = \frac{1.4}{20}$$

$$\therefore \boxed{p = 0.07}$$

$$\therefore P(X=2) = \binom{20}{2} (0.07)^2 (0.93)^{18}$$

$$\therefore \boxed{P(X=2) = 0.252}$$

$$(ii) \quad P(X \geq 1) = 1 - P(X=0) \\ = 1 - (0.93)^{20} = 0.766$$

$$\boxed{P(X \geq 1) = 0.766}$$

$$(iii) \quad P(s) = \text{probability of a box having at least one cracked egg} = 0.766$$

$Y = \#$ of boxes that have at least one cracked egg

probability that out of n boxes each/every box has at least one cracked egg $= P(Y=n) < 0.01$

$$\therefore P(Y=n) = \binom{n}{n} (0.766)^n (0.234)^0 \\ = (0.766)^n$$

$$\therefore (0.766)^n < 0.01$$

$$n \ln(0.766) < \ln(0.01)$$

$$n > 17.3$$

$$\therefore \boxed{n = 18}$$

$$(4)(i) \quad p(\text{success}) = \text{Probability Janice buys an item in any given week} \\ = 0.35$$

$$n = 10$$

$X = \#$ of weeks Janice buys an item.

$$\begin{aligned} P(X \leq 7) &= 1 - P(X \geq 8) \\ &= 1 - [P(X=8) + P(X=9) + P(X=10)] \\ &= 1 - \left[\binom{10}{8} (0.35)^8 (0.65)^2 + \right. \\ &\quad \left. \binom{10}{9} (0.35)^9 (0.65)^1 + \right. \\ &\quad \left. \binom{10}{10} (0.35)^{10} (0.65)^0 \right] \end{aligned}$$

$$\therefore \boxed{P(\text{at most } 7) = P(X \leq 7) = 0.995}$$

(ii) probability that Janice buys at least 1 item : $P(X \geq 1)$

$$P(X \geq 1) = 1 - P(X=0) > 0.99$$

$$\therefore 1 - \binom{n}{0} (0.35)^0 (0.65)^n > 0.99$$

$$1 - (0.65)^n > 0.99$$

$$-(0.65)^n > -0.01$$

$$(0.65)^n < 0.01$$

$$n \ln(0.65) < \ln(0.01)$$

$$-0.431n < -4.605$$

$$n > 10.7$$

$\therefore$ Smallest value of n is:

$$\boxed{n = 11}$$

(5) (i)

$P(\text{success}) = \text{Probability someone rates logo as good.}$
 $= 0.42$

$$n = 10$$

$X = \# \text{ people who rate logo as good}$

$$\begin{aligned} P(X < 8) &= 1 - P(X \geq 8) \\ &= 1 - [P(X=8) + P(X=9) + P(X=10)] \\ &= 1 - \left[\binom{10}{8} (0.42)^8 (0.58)^2 + \right. \\ &\quad \left. \binom{10}{9} (0.42)^9 (0.58)^1 + \right. \\ &\quad \left. \binom{10}{10} (0.42)^{10} (0.58)^0 \right] \end{aligned}$$

$$\therefore \boxed{P(X < 8) = 0.983}$$

(ii)

$$P(X \geq 1) > 0.995$$

$$1 - P(X=0) > 0.995$$

$$1 - \binom{n}{0} (0.42)^0 (0.58)^n > 0.995$$

$$1 - (0.58)^n > 0.995$$

$$-(0.58)^n > -0.005$$

$$\therefore (0.58)^n < 0.005$$

$$\ln (0.58)^n < \ln (0.005)$$

$$n \ln (0.58) < \ln (0.005)$$

$$-0.546n < -5.298$$

$$\therefore n > 9.7$$

$\therefore$ smallest value of n :

$$\boxed{n = 10}$$

Probability Mix

$$(1) \quad P(\text{Backpack}) = 0.65 \quad n = 12$$

(i) $X = \#$ of passengers that have a backpack. $p(s) = 0.65$

$$\begin{aligned} P(8 \leq X \leq 10) &= P(X=8) + P(X=9) + P(X=10) \\ &= \binom{12}{8} (0.65)^8 (0.35)^4 + \\ &\quad \binom{12}{9} (0.65)^9 (0.35)^3 + \\ &\quad \binom{12}{10} (0.65)^{10} (0.35)^2 \end{aligned}$$

$$\therefore \boxed{P(8 \leq X \leq 10) = 0.541}$$

$$(ii) \quad P(B) = 0.65$$

(Probability of
a backpack)

$$P(B') = 0.35$$

(Probability of
no backpack)

- Only possible way for this to is if the first 3 people don't have a backpack and 4th does.

$$P(B'B'B'B) = (0.35)^3 (0.65)$$

$$\therefore \boxed{P(B'B'B'B) = 0.279}$$

$$(2) \quad P(\text{Giraffes}) = P(G) = 0.90, \quad P(\text{zebras}) = P(Z) = 0.85 \\ P(\text{Elephants}) = P(E) = 0.95, \quad P(\text{Lions}) = P(L) = 0.1$$

$$(i) \quad P(\text{All}) = P(G) \cdot P(E) \cdot P(Z) \cdot P(L) \\ = (0.9)(0.95)(0.85)(0.10) \\ = 0.073$$

$$\boxed{P(\text{All}) = 0.073}$$

$$(ii) \quad n = 12 \quad X = \# \text{ of people that see Lions}$$

$$p(\text{success}) = P(L) = 0.1$$

$$\therefore P(X < 3) = P(X=0) + P(X=1) + P(X=2) \\ = (0.9)^{12} + \binom{12}{1}(0.1)(0.9)^{11} \\ + \binom{12}{2}(0.1)^2(0.9)^{10}$$

$$\boxed{P(X < 3) = 0.889}$$

$$(iii) \quad n = 50 \quad p(\text{success}) = P(Z) = 0.85$$

For a binomial distribution:

$$\mu = np = (50)(0.85) = 42.5$$

$$\sigma^2 = np(1-p) = (50)(0.85)(0.15) = 6.375$$

$$\boxed{\mu = 42.5}$$

$$\boxed{\sigma^2 = 6.38}$$

$$(3) \quad P(1) = P(2) = P(3) = 1/3 \quad n = 4$$

$$(i) \quad X = \# \text{ of scores that are } 3. \quad P(\text{success}) = P(3) = 1/3$$

$$\therefore P(X \geq 2) = 1 - P(X < 2) \\ = 1 - [P(X=0) + P(X=1)]$$

$$\therefore P(X \geq 2) = 1 - \left[\binom{4}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 + \binom{4}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 \right]$$

$$\therefore \boxed{P(X \geq 2) = 0.407}$$

$$(ii) \quad P(\text{sum is } 5) = ? \quad n = 4$$

$$\left. \begin{array}{cccc} 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 \end{array} \right\} \begin{array}{l} \text{Only ways} \\ \text{the scores} \\ \text{can sum to} \\ 5. \end{array}$$

$$P(\text{sum is } 5) = (4) (P(1))^3 (P(2))^1 \\ = 4 \left(\frac{1}{3}\right)^4 \\ = 0.049$$

$$\boxed{P(\text{sum is } 5) = 0.049}$$

$$(4) \quad P(\text{Rice}) = 0.75, \quad P(\text{Potato}) = 0.15, \quad P(\text{Pasta}) = 0.1$$

(i) $X = \#$ of days she eats pasta

$$n = 7$$

$$P(\text{success}) = P(\text{Pasta}) = 0.1$$

$$P(X=2) = \binom{7}{2} (0.1)^2 (0.9)^5$$

$$\boxed{P(X=2) = 0.124}$$

(ii) $R = \text{Rice}$, $P = \text{potato}$, $p = \text{pasta}$

"RRPpp" (Rice 2 days, potato 1 day, pasta 2 days)

* How many ways can "RRPpp" be arranged?

Accounting for RR $\frac{5!}{2!2!}$ ← Total ways different objects can be arranged.

Accounting for pp

$$\begin{aligned} \therefore P(\text{RRPpp}) &= \frac{5!}{2!2!} \times [P(\text{Rice})]^2 [P(\text{potato})] [P(\text{pasta})]^2 \\ &= \frac{5!}{2!2!} (0.75)^2 (0.15) (0.1)^2 \end{aligned}$$

$$\therefore \boxed{P(\text{RRPpp}) = 0.0253}$$

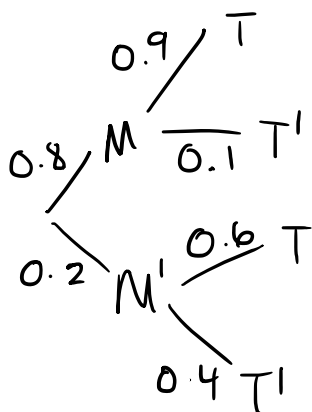
$$(5) \quad P(\text{Completes}) = 0.75$$

(i) $X = \#$ times he will complete the puzzle. $n = 5$
 $p(s) = 0.75$

$$\begin{aligned} P(X \geq 3) &= P(X=3) + P(X=4) + P(X=5) \\ &= \binom{5}{3} (0.75)^3 (0.25)^2 + \\ &\quad \binom{5}{4} (0.75)^4 (0.25) + \\ &\quad \binom{5}{5} (0.75)^5 (0.25)^0 \end{aligned}$$

$$\therefore \boxed{P(X \geq 3) = 0.896}$$

(ii)



M : Completes on Monday.

M' : Doesn't complete on Monday.

T : Completes on Tuesday

T' : Doesn't complete on Tuesday.

Probability he completes the puzzle on at least one of the two days.

possibilities: $P(MT)$, $P(MT')$, $P(M'T)$

$$\begin{aligned} P(\text{TOTAL}) &= P(MT) + P(MT') + P(M'T) \\ &= 1 - P(M'T') \end{aligned}$$

$$\begin{aligned}\therefore P(\text{TOTAL}) &= 1 - (0.2)(0.4) \\ &= 0.92\end{aligned}$$

$$\therefore \boxed{P(\text{TOTAL}) = 0.92}$$

(6)(i) $n=30$ $\mu=6.21$ $\mu=np$ — Probability of throwing 4.
 $\therefore p = \frac{\mu}{n} = \frac{6.21}{30}$

$$\therefore \boxed{p(\text{throwing } 4) = 0.207}$$

(ii) $\text{Var}(X) = np(p-1)$ $X = \# 4\text{'s thrown.}$
 $= (30)(0.207)(0.793)$
 $\boxed{\text{Var}(X) = 4.92}$

(iii) $p(s) = p(\text{throwing } 4) = 0.207$
 $n=15$

$$\begin{aligned} p(X \geq 2) &= 1 - p(X < 2) \\ &= 1 - [p(X=0) + p(X=1)] \\ &= 1 - \left[(0.793)^{15} + \binom{15}{1} (0.207)^1 (0.793)^{14} \right] \end{aligned}$$

$$\boxed{p(X \geq 2) = 0.848}$$

(7) $X = \text{Score when die is thrown.}$ $n = 3$

$$P(X = 4) = p$$

As sum of all probabilities is One :

$$4p + 0.2 + 0.2 = 1$$

$$4p = 0.6$$

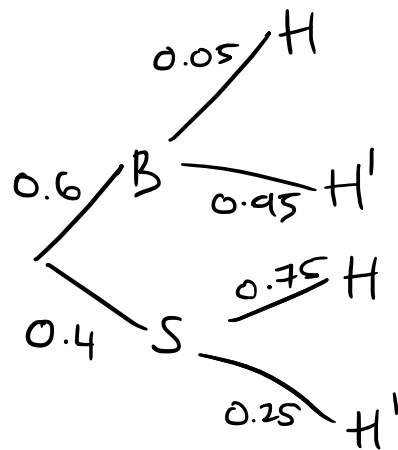
$$\boxed{p = 0.15}$$

$Y = \# \text{ of times the score is } 4.$ $p(5) = 0.15$

$$\begin{aligned} P(Y \leq 1) &= P(Y=0) + P(Y=1) \\ &= (0.85)^3 + \binom{3}{1}(0.15)^1(0.85)^2 \end{aligned}$$

$$\boxed{P(Y \leq 1) = 0.939}$$

(8) $P(\text{Bike}) = P(B) = 0.6$, $P(\text{skate}) = P(S) = 0.4$
 $H = \text{Hurts himself}$ $H' = \text{Doesn't Hurt himself.}$



$$\begin{aligned} \text{(i)} \quad P(\text{Hurts himself}) &= P(BH) + P(SH) \\ &= (0.6)(0.05) + (0.4)(0.75) \\ &= 0.33 \end{aligned}$$

$$\boxed{P(\text{Hurts himself}) = 0.33}$$

$$\begin{aligned} \text{(ii)} \quad P(S|H) &= \frac{P(S \text{ and } H)}{P(H)} && \text{(from formula)} \\ &= \frac{(0.4)(0.75)}{(0.33)} \end{aligned}$$

$$\therefore \boxed{P(S|H) = 10/11}$$

$$\begin{aligned}
 \text{(iii)} \quad n &= 45 & V(X_B) &= n p(B)(1-p(B)) \\
 & & &= (45)(0.6)(0.4) \\
 & & &= 10.8
 \end{aligned}$$

$$\begin{aligned}
 V(X_S) &= n p(S)(1-p(S)) \\
 &= (45)(0.4)(0.6) \\
 &= 10.8
 \end{aligned}$$

$$\therefore \boxed{V(X_B) = V(X_S) = 10.8}$$

$$\text{(iv)} \quad p(\text{success}) = p(\text{skate}) = 0.4 \quad n = 10$$

X = # of days Khalid rides his skateboard.

$$\begin{aligned}
 P(X \geq 2) &= 1 - P(X < 2) \\
 &= 1 - [P(X=0) + P(X=1)] \\
 &= 1 - [(0.6)^{10} + \binom{10}{1}(0.4)^1(0.6)^9] \\
 &= 0.954
 \end{aligned}$$

$$\therefore \boxed{P(X \geq 2) = 0.954}$$